# Quantitative Methods

## Multiple Regression

1. The General Multiple Linear Regression Model is:
   $Y_i = b_o + b_1 X_{1i} + b_2 X_{2i} + \ldots + b_k x_{ki} + \varepsilon_i$
2. **Hypothesis Testing of Regression Coefficient: -**
   $t = \frac{b_j - b_i}{s_{\hat{b}_j}}$ (n – k – 1) degrees of freedom
3. P-value: Smallest level of significance for which the null hypothesis can be rejected.
4. P Value > α; $H_0$ accept (α = Significance level)
5. P Value < α; $H_0$ rejected
6. **Confidence Interval**:
   $b_j \pm (t_c * s_{\hat{b}_j})$
7. SSE = $\sum(y_i - \hat{y})^2 = \sum_i^2$
   $\sigma\varepsilon^I = \sigma(y_i - \hat{y})$
   ↑
   SEE/SER = $\sqrt{\frac{\sum(x-\bar{x})^2}{n-2}} = \sqrt{\frac{\sum(\varepsilon_i - \overline{\varepsilon_i})^2}{n-2}}$
   If $\overline{\varepsilon_i}$=0; then:
   $= \sqrt{\frac{\sum\varepsilon_i^2}{n-2}} = \sqrt{\frac{SSE}{n-2}} = \sqrt{MSE}$
8. **Confidence Interval for A Regression Coefficient:**
   $\hat{b}_j \pm \left(t_c \times s_{\hat{b}_j}\right)$
   ↓
   standard error of $b_i$
   use (n-k-1) $d_f$
   where K = no of independent variables
   - $S_{bI} = f_n$ [SEE]
9. **Confidence interval for $\hat{y}$ -> forecasted variables**:
   Y = $\hat{Y} \pm (t_c \times s_f)$
   ↓
   S.E of $\hat{y}$
   TSS = SSE + ESS
   SSR RSS
   $\sum(yi - \bar{y})^2 = \sum(yi - \hat{y})^2 + \sum(\hat{y} - \bar{y})^2$
   ↓
   Can't explain
   ESS / RSS = the difference that is explained by independent variable.
   $R^2 = \frac{ESS}{TSS} = \frac{TSS-SSR}{TSS} = 1 - \frac{SSE/SSR}{TSS}$
   ↓
   Always +ve

10. **Predicting the dependent variable:**

$\hat{Y}_i = \hat{b}_0 + \hat{b}_1\hat{X}_{1i} + \hat{b}_2\hat{X}_{2i} + \cdots + \hat{b}_k\hat{X}_{ki}$

11. **The F - Statistics:**

F- stat = $\frac{\text{MSR}}{\text{MSE}}$ (one - tailed test)

(RSS/ESS)

Where MSR = mean regression sum of squares

MSE = mean squared error

(SSR/SSE)

12. **Coefficient of Determination:**

$R^2 = \frac{\text{RSS}}{\text{TSS}} = \frac{\text{ESS}}{\text{TSS}} = \frac{\text{TSS}-\text{SSE}}{\text{TSS}}$

13. **Adjusted $R^2$:**

$R_a^2 = 1 - \left[\left(\frac{n-1}{n-k-1}\right) \times (1 - R^2)\right]$

**Anova Table:**

1. $R^2 = \frac{\text{RSS}}{\text{TSS}}$

F = $\frac{\text{MSR}}{\text{MSE}}$with k and n-k-1 degrees of freedom

MSR = $\frac{\text{RSS}}{K}$ = $\frac{\text{RSS}}{1}$; K =1 for simple linear regression

MSE = $\frac{\text{SSE}}{n-K-1}$ = $\frac{\text{SSE}}{n-2}$

If K ≥ 1; R2 > Ra2 (No comparison of R2 & Ra2)

Ra2 = 1- [($\frac{n-1}{n-k-1}$) (1-$R^2$ )]

## Time-Series Analysis

1. Linear Trend Model:

$Y_t = b_0 + b_1 (t) + \varepsilon_t$

2. Ordinary Least Squares (OLS) regression:

$\hat{Y}_i = \hat{b}_0 + \hat{b}_1(t)$

3. Log - Linear Trend Model:

$Y_t = e^{b_0 + b_1(t)}$

4. Autoregressive Model (AR):

$X_t = b_0 + b_1X_{t-1} + \varepsilon_t$

5. AR (p) Model:

$X_t = b_0 + b_1X_{t-1} + b_2X_{t-2} + b_pX_{t-p} \ldots\ldots + \varepsilon_t$

6. Autocorrelation & Model Fit:

$t = \frac{\rho(e_t, e_{t-k})}{\frac{1}{\sqrt{T}}}$

7. Random Walk with A Drift:

   $X_t = b_0 + b_1 X_{t-1} + \varepsilon_t$

8. Covariance Stationarity:

   $X_t = b_0 + b_1 X_{t-1} + \varepsilon_t$

9. Unit Root Testing for Non-Stationary:

   $X_t = b_0 + b_1 X_{t-1} + \varepsilon$

   $X_t - X_{t-1} = b_0 + b_1 X_{t-1} - X_{t-1} + \varepsilon$

10. First Differencing:

    $Y_t = X_t - X_{t-1} \Rightarrow Y_t = \varepsilon_t$

    $Y_t = b_0 + b_1 Y_{t-1} + \varepsilon_t$

11. ARCH (1) Regression Model:

    $\hat{\varepsilon}_t^2 = a_0 + a_1 \hat{\varepsilon}_{t-1}^2 + \mu_t$

12. Predicting the Variance of a Time Series:

    $\hat{\sigma}_{t+1}^2 = \hat{a}_0 + \hat{a}_1 \hat{\varepsilon}_t^2$

**Problems:**

| | | | | | |
|---|---|---|---|---|---|
| | Hetero-skedasticity | Meaning<br>$\sigma_{\varepsilon i}$ not constant<br>Conditional<br>Based on 'x'<br>unconditional<br>random | Effect<br>$b_I$ not affected<br>F-test X<br>$S_{bI}$ X<br>t-stat – inaccurate | Detection<br>Scatter plot<br>Breush pagan (BP) test<br>$n \times R^2$ | Correction<br>Robust SE/ white corrected SE<br>Generalized least squares |
| 1 | Serial correlation | Errors terms are correlated<br>$\rho Ei, Ej \neq 0$ | $S_{bI}$ x ↓; t stat ↑<br>F-test x<br>Type I error | Residual plot → the $S_{bI}$ ↓<br>Type I error → -ve $S_{bI}$↑ Type II Error<br>$DW_{stat} = 2(I-r)$<br>$H_a$ : positively correlated<br>$H_0$ : not positively correlated | CH+ serial = Hansen<br>Only serial = Hansen<br>Improve specification of model |
| 1 | Multi collinearity | $r_{XI, X2} \neq 0$ | $S_{bI}$ ↑ x<br>$b_I$, $b_2$ unaffected<br>∴ Prob of type II Error ↑ | If individual coefficients are insignificant but F-test, $R^2$ is significant then $x_I$ $x_2$ correlation ↑<br>Multi collinearity | • Omit correlation independent variable<br>• Stepwise regression |

# Economics

## Currency Exchange Rates: Understanding Equilibrium Value

1. Mark To Market Value = $\frac{\text{(Forward Price New} - \text{F.P locked old)(Contract Size)}}{[1+r\left(\frac{n}{360}\right)]}$

   $\downarrow$ Price currency

2. Covered Interest Rate Parity:

   $F_{A/B} = S_{A/B}\left(\frac{1+iA}{1+iB}\right)$

   F (disc/premium) = $S_A + \left(\frac{(iA-iB)\left(\frac{n}{360}\right) \rightarrow \text{if LiBOR rates}}{(1+iB)\ n/360)}\right)$

3. Uncovered Interest Rate Parity:

   $E(\%\,\Delta S)_{(A/B)} = R_A - R_B$

4. International Fisher Relation:

   $R_{nominal\ A} - R_{nominal\ A} = + E(Inflation_A) - E(Inflation_B)$

5. Purchasing Power Parity:

   $F = S\left(\frac{1+Inflation_A}{1+Inflation_B}\right)$

6. Absolute PPP:

   $S(A/B) = \frac{CPI\ (A)}{CPI(B)}$

7. Relative PPP:

   $\%\,\Delta\, S\,(A/B) = Inf_A - Inf_B$ $CPI_B$

8. Real Exchange Rate = $S_t \times \frac{(1+r_B)}{(1+r_A)}$ $T$ CPI

9. BOP ⇒ Current A/c + Capital A/c + Official Reserve A/c = 0

10. Real exchange rate A/B = (Equilibrium Real Exchange Rate A/B)

    (B affricates) ↑+ (↑Real Int. $rate_B$ – Real Int. $rate_A$)

    \- (Risk $Premium_B$ - ↑Risk $Premium_A$)

11. Taylor's Rule: -

    $R = r_n + \pi + \alpha(\pi-\pi^*) + \beta(y-y^*)$

12. Real Interest Rate = $r_n + \pi + \alpha(\pi - \pi^*) + \beta(\gamma-\gamma^*)$

## Economic Growth

1. $\Delta P = \Delta GDP + \Delta(E/GDP) + \Delta(P/E)$
2. Cobb – Douglas Function:

   $Y = TK^{\alpha} L^{(1-\alpha)}$
3. Output Per Worker = $Y/L = T(K/L)\alpha$
4. Marginal Product of Capital = $\frac{\Delta Y}{\Delta K} = \frac{\alpha Y}{K}$

   (Constant)

   Marginal Productivity = $\frac{\Delta Y/L}{\Delta K/L}$, K↑ L = constant

   (Diminishing)
5. MP K = r (Marginal cost of K)-> rental price of capital

   ↓
6. Growth Accounting Relation:

   $\Delta Y/Y = \Delta A/A + \alpha (\Delta K/K) + (I\text{-}\alpha) (\Delta L/L)$
7. Growth in Potential GDP = i) Long Term Growth of Technology + α (Long Term Growth of K) + (I-α) (Long-Term Growth of L)

   ii) Long Term Growth of Labour Force + Long Term Growth in Labour Productivity (Output Per Worker) Both Capitals Depending + Technology Process.
8. Labour Force Participation = $\frac{\text{Labour Force}}{\text{Working Age Population}}$

   Where Labour Force = Employed + Unemployed Available to Work.
9. G* (Growth of Output Per Capita) = $\frac{\theta}{I-\alpha}$
10. G* (Growth of Output) = $\frac{\theta}{I-\alpha} + \Delta L$

# Financial Statement Analysis

## Intercorporate Investments

1. Full Goodwill = (Fair Value of Equity of Whole Subsidiary) – (Fair Value of Net Identifiable Assets of The Subsidiary)
   - Allowed under both IFRS & USGAAP
2. Partial goodwill = Purchase Price – (% owned X FV of Net Identifiable Assets)
   - Allowed under only IFRS
3. Goodwill Impairment

US GAAP
1) CA > FV of reporting unit
2) CA of g/w –implied FV of g/w
   ↓
   FV of unit – net identifiable asset

IFRS
1) CA> RA => loss in P/L

Years to Repay Debt from CFO = $\frac{\text{Total Debt}}{\text{Operating CF} - \text{Reinvestment}}$

## Employee Compensation: Post-Employment and Share-Based

1. Pension

Plan Asset Plan Assets

- FV at beginning of year
  + Contributions
  + Actual Return
  $-$ Benefits paid
  $=$ FV at end of year

PBO at beginning of year
(+) Current service cost
(+) Interest cost
(+) PSC
(+) Actual Loss/ (-) gain
$(-)$ Benefits
$=$ PBO at end of year

2. Plan Asset > PBO – Overfunded Plan
3. Plan Asset < PBO – Underfunded Plan
4. Funded Status = Fair Value of Plan Asset - PBO
5. TPPC = Employer contributions – [Ending Funded status – Beginning Funded status]
   = Employer contribution – [(End Plan Asset – End PBO) – (Beginning plan Asset – Beginning PBO)]
   = Employer contribution – [(Ending Plan Asset – Beginning Plan Asset) – (Ending PBO - Beginning PBO)]
   = ~~Cont^n^~~ – [(~~cont^n^~~+ Actual Return – ~~Benefit~~) - (Current SC + Int cost + PSC $\pm$ Acturial gain/ loss- ~~Benefit~~)
   TPPC = Current SC + Int cost + Past SC $\pm$ Actuarial gain / loss – Actual Return

US. GAAP
v. Current Service cost in P/L
vi. Interest cost in P/L
    Expected Return in P/L
vii. PSC in OCI
    (amortized over remaining life)
viii. Actuarial g/l in OCI
    (expected return – actual return)
    Corridor approach
    ↓
    Amortize amount that is above 10% x max (A, L) over the remaining life

IFRS
i. Current Service cost in P/L
ii. Net Interest income in P/L
    -income if overfunded (A > L)
    -expense if underfunded (A < L)
iii. PSC in P/L
    (recognized immediately)
iv. Actuarial g/l in OCI
    (expected return actual return)
    ↓
    Not amortized over

6. Expected Return = Expected rate x Beginning plan Asset
7. Net Int cost/ income = Disc. Rate x Beginning funded status (A-L)
8. For IFRS, disc rate & expected return is same
9. Periodic pension cost in OCI = TPPC – periodic pension cost in P/L

   Or

   Actuarial Gains/ losses +(Actual – expected) return
10. To reclassify:
    - Op. income + Full pension exp – current SC
    - Add int cost to int exp.
    - Add Actual return to other (non -operating) income.
11. Cont$^n$ > TPPC -> reduction in PBO

    Cont$^n$ < TPPC => source of borrowing

    From CFO -> (+)

    CFF -> (-)

## Evaluating Quality of Financial Reports

1. The Beneish Model (M-score):

   i. DSRI: $\frac{\text{Days Rec}_t}{\text{Days Rec}_{t-1}}$ ↑ X

   ii. GMI: $\frac{\text{Gross Margin}_{t-1}}{\text{Gross Margin}_t}$ ↑ X

   iii. AQI: $\frac{\text{Noncurrent asset except PP \&ET}_t / \text{Total Assets}_t}{\text{NCA except PP \&ET}_{t-1} / \text{Total Assets}_{t-1}}$ ↑ X

   iv. SGI: $\frac{\text{Sales}_t}{\text{Sales}_{t-1}}$ ↑ X

   v. DEP: $\frac{\text{Depreciation}_{t-1}}{\text{Depreciation}_t}$ ↑ X

   vi. SGAI: $\frac{\%\text{SGA/sales}_t}{\%\text{SGA/sales}_{t-1}}$ ↑ X

   vii. Accruals: $\frac{\text{Income Before EOI} - \text{CFO}}{\text{Assets}}$ ↑ X

   viii. Leverage Index: $\frac{\text{D/A}_t}{\text{D/A}_{t-1}}$ (*higher the better↓) X

2. Gauging Earning Persistence:

   $$\text{Earnings}_{(t+1)} = \alpha + \beta_1 \text{earnings}_t + \varepsilon$$

   $$\text{Earnings}_{(t+1)} = \alpha + \beta_1 \text{cash flow}_t + \beta_2 \text{accruals}_t + \varepsilon$$

## Integration of Financial Statement Analysis Techniques

1. Sources of Earnings and ROE:

   DuPont Decomposition -

   ROE = NI/EBT x EBT/EBIT x EBIT/Revenue x Revenue/Average Asset X Average Asset/Average Equity

2. Accruals BS = NOA End – NOA Bgn

3. Accruals Ratio BS = $\frac{\text{Accruals}^{\text{BS}}}{(\text{NOA}_{\text{End}} + \text{NOA}_{\text{Beg}})/2}$

4. Accruals CF = NI – CFO –CFI

5. Accruals ratio CF = $\frac{\text{Accruals}^{\text{CF}}}{(\text{NOA}_{\text{End}} + \text{NOA}_{\text{Beg}})/2}$

6. CGO = EBIT + non-cash changes – increase in WC

| | IFRS | USGAAP |
|---|---|---|
| Int paid | CFO/ CFF | CFO |
| Div. Paid | CFO/ CFF | CFF |
| Int/Div Recd. | CFO/CFI | CFO |

- Market Value decomposition

  Implied value = Parent's MV – Parents Pro- rate share in associate's MV

## Financial Statement Modeling

1. Cost of Goods Sold (COGS):

   Forecast COGS = (Historical COGS/revenue) x (Estimate of Future Revenue)

2. Financing Cost:

   Net debt = Gross debt – Cash, cash equivalents &short-term investment.

   Net interest expense = Gross Interest Expense – Interest Income (on cash & short- term debt securities)

3. Gross (net) interest expense rate = gross (net) expense/ gross (net) debt

4. Yield on average cash = interest income / cash + ST securities.

5. Effective tax rate = $\frac{\text{Income tax expense}}{\text{PBT}}$

   Cash tax rate = $\frac{\text{Cash taxes paid}}{\text{PBT}}$

6. Projected Accounts Receivables = Days Sales Outstanding x $\left(\frac{\text{Forecasted Sales}}{365}\right)$

7. ROC = $\frac{\text{Net Operating Profit}}{\text{D+E}}$ → Not adjusted for taxes

   ROIC = $\frac{\text{NOPLAT}}{\text{D+E}}$ → Net operating profit - taxes

   ROE = $\frac{\text{NI}}{\text{E}}$ → Not suitable for comparing companies with different capital structures

8. Cannibalization rate = $\frac{\text{new product sales that replace existing product sales}}{\text{total new product sales}}$

# Corporate Issuers

## Analysis of Dividends and Share Repurchases

1. **Effective tax rate** = corporate tax rate + (1 - corporate tax rate) (individual tax rate)

2. **Expected increase in dividends** = [(expected earnings × target payout ratio) - previous dividend] adjustment factor

3. **FCFE coverage ratio** = FCFE / (dividends + share repurchases)

4. **Grinold-Kroner model:** $\text{ERP} = [DY + \Delta P/E + i + G - \Delta S] - r_f$

5. **Cost of equity based on DDM:** Cost of equity ($r_e$) = dividend yield (DY) + capital gains yield (CGY)

6. **Fama-French model:** $Required\ return\ of\ stock = r_f + \beta_1 ERP + \beta_2 SMB + \beta_3 HML$

7. **Five-factor Fama-French extended model:** $Required\ return\ of\ stock = r_f + \beta_1 ERP + \beta_2 SMB + \beta_3 HML + \beta_4 RMW + \beta_5 CMA$

8. **Expanded CAPM for private companies:** $Required\ return = r_f + \beta_{peer} \times ERP + SP + IP + SCRP$

9. **Build-up approach:** $Required\ return = r_f + ERP + SP + SCRP$

10. **Premium Paid Analysis:** Premium = (DP-UP) / UP

    = (deal price - unaffected price) / unaffected price

11. **Gross profit margin** = $\frac{gross\ profit}{net\ sales}$

12. **Operating profit margin** = $\frac{operating\ profit}{net\ sales} = \frac{EBIT}{net\ sales}$

13. **Net profit margin** = $\frac{net\ income}{net\ sales}$

14. **Return on assets** = $\frac{net\ income}{average\ total\ assets}$

15. **Return on total capital** = $\frac{EBIT}{(interest\ bearing\ debt + shareholders'\ equity)}$

16. **Return on total equity** = $\frac{net\ income}{average\ total\ equity}$

17. **Financial leverage ratio** = $\frac{total\ assets}{total\ equity}$

18. **Long-term debt-to-equity ratio** = $\frac{total\ long\text{-}term\ debt}{total\ equity}$

19. **Debt-to-equity ratio** = $\frac{total\ debt}{total\ equity}$

# Equity

## Equity Valuation: Application & Processes

1. $IV_{analyst} - price = (IV_{actual} - price) + (IV_{analyst} - IV_{actual})$
2. Conglomerate Discount = Sum of individual parts – Sum as a whole.

## Discounted Dividend Valuation

DDM Model:

1. One Period -

   $P_0 = \frac{D_1+P_1}{I+R_e}$

2. Two period -

   $P_0 = \frac{D_1}{1+R_e} + \frac{D_2+P_2}{(1+R_e)^2}$

3. Multi period -

   $P_0 = \frac{D_1}{1+R_e} + \frac{D_2}{(1+R_e)^2} + \ldots + \frac{D_n+P_n}{(1+R_e)^n}$

4. Gordon Growth Model:

   $P_0 = \frac{D_1}{R_e - g}$

5. Present value of growth opportunities (PVGO):

   $P_0 = \frac{E_1}{R_e} + PVGO$

6. Justified trailing P/E:

   $\frac{P_0}{E_0} = \frac{(I+g)\,(I-b)}{R_e - g}$

   Justified leading P/E:

   $\frac{P_0}{E_1} = \frac{(I-b)}{R_e - g}$

7. Value of perpetual preferred shares = $\frac{D_p}{r_p}$

8. Valuation using H – Model

   $V_0 = \frac{D_0(1+g_L)}{R_e - g_L} + \frac{D_0 \times {}^t/_2 \times (g_S - g_L)}{R_e - g_L}$

9. Sustainable growth rate:

   SGR (g) = b x ROE

   Where, ROE = $\frac{NI}{Stockholders' Equity} = \frac{NP(NI)}{sales} \times \frac{Sales}{Total Assets} \times \frac{Total Assets}{Stockholders' Equity}$

   $\therefore g = \left(\frac{Net Income - Dividends}{Net Income}\right) \times \frac{Net Income}{Sales} \times \frac{Sales}{Total Asset} \times \frac{Total Assets}{Stockholders' Equity}$

10. Intrinsic value > market value → undervalued
11. Intrinsic value = market value → fairly valued
12. Intrinsic value < market value → overvalued

## Free Cash Flow Valuation

1. **FCFF – 4 Approaches:**
   i. FCFF = NI + NCC + Interest (1 – TAX) - fC Investment - WC Investment
   ii. FCFF = **NI + NCC – WC Investment** + Interest (1 – TAX) - fC investment
   = **CFO** + Interest (1 – TAX) - FC Investment
   iii. FCFF = EBIT (1 – TAX) + Depreciation - fC investment - WC investment
   iv. FCFF = EBITDA (1 – TAX) + (Depreciation X TAX) - fC investment - WC investment
2. **FCFE – 4 Approaches:**
   FCFE = FCFF - Interest (1 – TAX) + NET Borrowings
   FCFE = NI + NCC - WC Investment - FC Investment + Net Borrowings
   FCFE = CFO - FC Investment + Net Borrowings
   FCFE = NI – (1- DR) [(FC Investment – Depreciation) + WC Investment]
   $\therefore$ $1 - DR = 1 - \frac{D}{A} = \frac{A-D}{A} = \frac{E}{A}$
3. **Single Stage FCFF / FCFE Model:**
   FCFF: Value of Firm = $\frac{FCFF_1}{WACC-g} = \frac{FCFF_0 \times (1+g)}{WACC-g}$
   FCFE: Value of Equity = $\frac{FCFE_1}{r-g} = \frac{FCFE_0 \times (1+g)}{r-g}$
4. Terminal Value in year n= (trailing P/E) x (earnings in years n)
5. Terminal Value in year n = (leading P/E) x (forecasted earnings in year n +1) = $\frac{P}{E_1}$x $E_O(1+g)$

## Market-Based Valuation: Price and Enterprise Value Multiples

1. Trailing P/E = $\frac{\text{Market price per share}}{\text{EPS over previous 12 months}}$
2. Leading P/E = $\frac{\text{Market price per share}}{\text{Forecasted EPS over next 12 months}}$
3. P/B Ratio = $\frac{\text{Market price of Equity}}{\text{Book value of Equity}}$
4. P/S Ratio = $\frac{\text{Market value of Equity}}{\text{Total Sales}}$
5. Dividend Yield: D/P
6. Trailing D/P = $D_0/P_0$
7. Leading D/P = $D_1/P_0$
8. Justified P/E Multiple:

   $P_0 = \frac{D_1}{R_e - g}$

   Justified trailing P/E = $\frac{P_0}{E_0} = \frac{(1-b)(1+g)}{r-g}$

   Leading P/E = $\frac{P_0}{E_1} = \frac{1-b}{r-g}$
9. Justified P/B Multiple:

   Justified P/B Ratio = $\frac{ROE-g}{r-g}$
10. Justified P/S Multiple:

    Justified $\frac{P_0}{S_0} = \frac{\left(\frac{E_0}{S_0}\right)\times(1-b)(1+g)}{r-g}$
11. Justified P/CF Multiple:

    $V_0 = \frac{FCFE_0(1+g)}{r-g}$
12. Justified Dividend Yield:

    $\frac{D_0}{P_0} = \frac{r-g}{1+g}$

13. FED & YARDENI Model:

    Fed model:

    $(E/P)_{S\&P} > (E/P)_{10\text{ yr T-Bond}}$ $\therefore$ undervalued

    $(E/P)_{S\&P} < (E/P)_{10\text{ yr T-Bond}}$ $\therefore$ overvalued

    Yardeni model:

    Earnings yield of market (E/P) = yield on 'A' rated bond – k x (Long term growth rate)
14. PEG RATIO:

    Peg ratio = $\frac{\text{P/E ratio}}{g}$

    CF = Net Income + depreciation + amortization

    FCFE = CFO – FC Inv + Net borrowing

    P/CF = $\frac{\text{MV of equity}}{\text{CF}} = \frac{\text{Market price per share}}{\text{CF Per Share}}$

    EV/EBITDA $_{\text{ratio}}$ = $\frac{\text{enterprise value}}{\text{EBITDA}}$
15. Momentum indicator:

    i. Earnings Surprise = Reported EPS – Expected EPS

    ii. Standardized Unexpected Earnings (SUE) = $\frac{\text{earnings surprise}}{\text{SD of earnings surprise}}$
16. Weighted Harmonic Mean = $\frac{1}{\sum_{i=1}^{n}\frac{w_i}{X_i}}$

## Residual Income Valuation

1. RI = Net Income – Cost of Equity X Equity Capital (equity charge)
   = (ROE – r) $BV_{equity\ (t-I)}$ where BV = beginning BV
2. EVA = NOPAT – (WACC x TOTAL CAPITAL) → Beginning invested capital (D + E)
   = [EBIT – (1-tax)] – WACC
3. MVA = Market Value -Total Capital
4. $RI_t = E_t - (r \times B_{t-1}) = (ROE - r) \times B_{t-1}$
5. Intrinsic Value:
   $$P_0 = B_0 + \left\{\frac{RI_1}{(1+r)^1} + \frac{RI_2}{(1+r)^2} + \frac{RI_3}{(1+r)^3} + \cdots\right\}$$
6. Single - stage Residual Model:
   $$P_0 = B_0 + \left[\frac{(ROE-r)\times B_0}{r-g}\right]$$
7. The growth rate implied by the market price in a single - stage residual income:
   $$g = r - \left[\frac{B_0\times(ROE-r)}{P_0 - B_0}\right]$$
8. Tobin's Q = $\frac{\text{market value of debt + market value of equity}}{\text{replacement cost of total asset}}$
9. $P_0 = B_0 +$ (PV of interim high-growth RI) + (PV of continuing residual income)
10. PV of Continuing Residual Income in year T – 1 = $\frac{RI_T}{1+r-\omega}$

## Private Company Valuation

1. $V_F = \frac{FCFF_1}{WACC-g}$
2. $V_E = \frac{FCFE_1}{K_e-g}$
3. Control premium = pro rata value of controlling interest –pro-rata value of non-controlling interest.
4. Adjusted control premium (applicable for MVIC multiple) = (control premium on equity) x (1 – DR) [DR = Debt to asset ratio]
5. DLOC = 1 – [1/ (1 + control premium)
6. Total discount for lack of marketability = 1 – [(1 – DLOC) (1 – DLOM)]

# Fixed Income

## The Term Structure & Interest Rate Dynamics

1. The Relationship Between the Discount Factor $P_T$ and the Spot Rate $S_T$:

   $P_T = \frac{1}{(1+S_T)^T}$

2. Forward Rates:

   $F_{(j,k)} = \frac{1}{[1+f(j,k)]^k}$

3. The Forward Pricing Model:

   $F_{(j,k)} = \frac{P_{(j+k)}}{P_j}$

4. The Forward Rate Model:

   $\left[1 + S_{(j+k)}\right]^{(j+k)} = (1 + S_j)^j [1 + f(j,k)]^k$

5. Swap Rate = $\frac{1-d_L}{\sum d_L}$

6. Swap $\text{Spread}_t$: = Swap $\text{Rates}_t$ – Treasury $\text{yield}_t$ (Same Maturity)

7. I spread = Riskiness of Corporate Bond Over Banks

   ↓

   $\text{comp}^n$ credit liquidity risk = corp Bond – swap rate

8. TED Spread = (3months LIBOR Rate) – (3months T-bill Rate)

9. LIBOR – O/S Spread = ↑ = banks unwilling to lend; ↓ = liquidity

   Includes credit risk | minimal credit risk | measure of money market securities risk

10. Cox – Ingersoll Ross Model (CIR):

    $dr = [a\,(b - r)\,dt + (\sigma\sqrt{r}.dz)]$

11. Vasicek Model:

    dr = [(a(b-r) dt + ( $\sigma$dz)]

12. Ho-Lee Model:

    $dr_t = \theta_t\, dt + \sigma dz_t$

13. Sensitivity to Parallel, Steepness, and Curvature Movements:

    $\frac{\Delta P}{P} = D_L \Delta x_L - D_S \Delta x_S - D_C \Delta x_C$

## Valuation and Analysis Bonds with Embedded Options

1. Value of option embedded in a bond:
   $V_{Call}$ = $V_{straight}$ bond – $V_{callable}$ bond
   $V_{Put}$ = $V_{Putable}$ bond – $V_{straight}$ bond.
2. OAS = Z – Call Risk
3. OAS = Z + Put Risk
4. Effective Duration = $\frac{P_2 - P_1}{2P_0 \Delta y}$
5. Effective Convexity = $\frac{P_2 + P_1 - 2P_0}{P_0(\Delta y)^2}$
6. Market Conversion Premium Ratio = $\frac{\text{Conversion Premium} \times \text{Market Per Share}}{\text{Market Price of Convertible Stock}}$
7. Conversion Value = Market Price of Stock x Conversion Ratio.
8. Market Conversion Price = $\frac{\text{Market Price of Convertible bond}}{\text{Convertion Ratio}}$
9. Market Conversion Premium Per Share = Market Conversion Price – Stock's Market Price.
10. Premium Over Straight Value = $\frac{\text{Market Price of Convertible Bond}}{\text{Straight Value}}$ -1
11. Put – Call Parity:
    C – P = PV (Forward price of the bond on exercise date) – PV (Exercise price)

## Credit Analysis Models

1. Present Value of Expected Losses = Expected Loss + Risk Premium – Time Value Discount
2. Value of $\text{Stock}_T$ = Max (0, $A_T$ -K)
3. Value of $\text{Debt}_T$ = Min ($A_T$, K)
4. Probability of Default = I - N ($e_2$)
   $e_1 = \frac{\ln\left(\frac{At}{K}\right) + \mu\,(T-t) + {}^{1}/_{2}\,\sigma^2\,(T-t)}{\sigma\sqrt{T-t}}$
   Where $\mu$ = Annual rate of return on company's assets.
   $e_2 = e_1 - \sigma\sqrt{T-t}$
5. Key rate duration total duration -> same effect if parallel shift
6. Duration exposure = Add the duration
7. Effective Duration = $\frac{P_2 - P_1}{2P_0 \Delta y}$
8. Effective Convexity = $\frac{P_2 - P_1}{P_0 (\Delta y)^2}$
9. % $\Delta$ Bond Price = $-\Delta y$ x ED = $\frac{1}{2}$ x EC x $(\Delta y)^2$
10. VCB = VNCB – Call Price
11. VPB = $VN_{PB}$ + Put Price

## Credit Default Swaps

1. Pay-out Amount = pay-out Ratio x NP
2. Pay-out Ratio = I – (Recovery Rate) %
3. Hazard rate/ conditional Prob. Of default = Prob. (PD/ Default has not occurred)
4. Expected Loss = Hazard Rate x LGD (% terms)
5. Upfront payment = $\underbrace{\text{PV (protection leg)}}_{\text{Based on CDS spread}} - \underbrace{\text{PV (premium leg)}}_{\text{Based on coupon rate}}$
6. Upfront Premium = (CDS spread – CDS coupon) x duration of spread
7. Price of CDS = 100 – Upfront Premium (%)
8. Valuation After Inception of CDS:
   Profit for protection buyer ≈ ($\Delta$spread x duration) x Notional Principal Or,
   Profit for protection buyer (%) ≈ change in spread (%) x duration

# Derivatives

## Pricing and Valuation of Forward Commitments

1. Forward Price = Price That Prevents Profitable Riskless Arbitrage in Frictionless Markets
2. The Forward Contract Price:

   FP = $S_0 \times (I+R_f)^t$
3. Forward Contracts with Discrete Dividends:

   FP (on An Equity Security) = (S0 – PVD) x (1+Rf) T

   FP (on an Equity Security) = $[S_0 \times (1+R_f)^T] - \text{FVD}$
4. Value of the Long Position in A Forward Contract on A Dividend Paying Stock:

   $V_t$ (long position) = $(S_t - PVD_t) - \left[\frac{\text{FP}}{(I+R_f)^{T-t}}\right]$
5. Equity Forward Contracts with A Continuous Dividends:

   FP (on An Equity Index) = $S_0 \times e^{(R_f^c - \delta^c)\times T}$ = $\left(S_0 \times e^{\delta^c \times T}\right) \times e^{R_f^c \times T}$

   F = $\frac{\text{Spot} \times e^{\text{interest} \times t}}{e^{\text{Dividends} \times t}}$
6. Forward Price on A Coupon Paying Bond:

   FP (on A Fixed Income Security) = $(S_0 - PVC) \times (1+R_f)^T = S_0 (1+R_f)^T$ - FVC
7. $V_t$ (Long Position) = $(S_t - PVC_t) - \left[\frac{\text{FP}}{(I+R_f)^{T-t}}\right]$

   Accrued Interest = $\frac{\text{Days Since Last Coupon}}{\text{Days Between Coupon Payment}}$ x Coupon Amount
8. FP = $\left[(\text{Full price})(1+R_f)^T - \text{AI}_T - \text{FVC}\right]$
9. QFP = FP/CF = $\left[(\text{Full Price})(1+R_f)^T - \text{AI}_T - \text{FVC}\right]\left(\frac{1}{\text{CF}}\right)$
10. Covered Interest Rate Parity:

    FT (Currency Forward Contract) = S0 $\times \frac{(1+R_{PC})^T}{(1+R_{BC})^T}$
11. Valuing Currency Forward Contracts After Initiation:

    $V_t = \frac{[FP_t - FP]\times(\text{contract size})}{(1+r_{PC})^{(T-t)}} = \left[\frac{S_t}{(1+R_{BC})^{T-t}}\right] - \left[\frac{F_T}{(1+R_{PC})^{T-t}}\right]$
12. Value of Futures Contract = Current Futures Price – Previous Mark to Market Price.
13. Discount factors (zs):

    $Z = \frac{1}{\left[1+\left(\text{LIBOR} \times \frac{\text{days}}{360}\right)\right]}$
14. SFR (Periodic) = $\frac{1 - \text{last discount factor}}{\text{sum of discount factors}}$
15. Market Value of An Interest Rate Swap:

    Value to The Payer = $\sum Z \times (SFR_{New} - SFR_{Old}) \times \frac{\text{days}}{360} \times \text{Notional Principal}$
16. Equity Swaps:

    SFR (Periodic) = $\frac{1 - \text{Last Discount Factor}}{\text{Sum of Discount Factors}}$

## Valuation of Contingent Claims

1. Put – Call Parity:

$C_0 + \frac{X}{(1+R_F)^T} = P_0 + S_D$

$H = \frac{C^+ - C^-}{S^+ - S^-}$

2. Black – Scholes Model:

$Co = S0\ N(d1) - e^{-rt}XN(d_2)$

$Po = e^{rt}XN(-d_2) - S0\ N(-d1)$

Where:

$d1 = \frac{\ln\left[\frac{S}{X}\right] + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$

$d2 = d1 - \sigma\sqrt{T}$

3. Options on Dividend Paying Stocks:

$Co = S0\ e^{-\delta T}N(d1) - e^{-rt}N(d_2)$

$P0 = e^{-rt}N(-d_2) - S0\ e^{-\delta T}N(-d1)$

Where $\delta$ = Continuously Compounded Dividend Yield

$d1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$

$d2 = d_1 - \sigma\sqrt{t}$

4. Options on Currencies:

$Co = S0\ e^{-rBT}N(d1) - e^{-rPT}N(d_2)$

$P0 = e^{-rPT}N(-d_2) - S_0\ e^{-rBT}N(-d_1)$

5. The Black Model: -

$Co = \frac{F_T}{e^{rt}}N(d_1) - \frac{X}{e^{rt}}N(d_2)$

Where, $d1 = \frac{\ln\left(\frac{F_T}{x}\right) + \left(\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$

$d2 = d_1 - \sigma\sqrt{t}$

6. Interest Rate Options:

$Co = \frac{AP}{e^{r\left(N \times \frac{30}{360}\right)}}$ [FRA (M*N) N (d1) – XN (d2)] X NP

Where: AP = Accrual Period $= \frac{\text{Actual}}{365} = \left[\frac{(N-M) \times 30}{360}\right]$

NP = Notional Principal on the FRA.

7. Swaptions:

$V_{\text{payer swaption}}$ = PVA [SFR. N ($d_1$) – XN ($d_2$)] x NP x AP

8. $\Delta C$ = Call Delta x $\Delta s$ + $\frac{1}{2}$ Gamma x $\Delta s2$

9. $\Delta P$ = Put Delta x $\Delta s$ + $\frac{1}{2}$ Gamma x $\Delta s^2$

# Alternative Investments

## Real Estate Investment

1. **Income Approach**

**Net Operating Income:**

Rental income (if fully occupied)
+ other income
=Potential gross income
- vacancy and collection loss
=Effective gross income
- Operating expense
=Net operating income

**The Capitalization Rate:**

1. Cap Rate = $\frac{NOI_I}{Value}$

   or, Value = $V_O = \frac{NOI}{Cap.Rate}$

   or, Cap Rate = $\frac{NOI}{\text{Comparable Sales Price}}$

$P_O = \frac{D_1}{Re-g}$

or, $P_O = \frac{NOI}{Cap.Rate}$

or, Cap Rate = $\frac{NOI}{P_O}$

$\downarrow$

Value of property today

2. Value = $\frac{Rent}{ARY}$

   or, ARY = $\frac{Rent}{\text{Comparable Sale } P_x}$

3. Gross Income Multiplier = $\frac{\text{Sales Price}}{\text{Gross Income}}$

4. Value = Gross Income × Gross Income Multiplier

   **Discounted Cash Flow Method:**

   1. Cap Rate = Discounted Rate – Growth Rate = $R_e$ - g
   2. Discounted Rate = Cap Rate + Growth Rate
   3. Value = $V_O$ = $\frac{NOI}{(r-g)} = \frac{NOI}{\text{Cap Rate}}$

2. **COST APPROACH**

$V_{property}$ = (Replacement cost – Cost of fixing curable items) – Depreciation – Incurable expense capitalised + Market value of land

Where Depreciation = $\frac{\text{Effective Age}}{\text{Economic Life}} \times [\text{Replacement Cost} - \text{Curable Items}]$

3. **Appraisal-Based indices:**

   a) Return = $\frac{\text{NOI – Capital Expenditure + End Market Value – Beginning Market Value}}{\text{Beginning Market Value}}$

4. Ratios to consider for evaluation

   a) Debt Service Coverage Ratio. = $\frac{\text{First year NOI}}{\text{Debt Service}}$ [higher the better]

   b) Loan-to - Value Ratio = $\frac{\text{Loan Amt}}{\text{Appraisal Value}}$ [lower the better]

   c) Equity Dividend Rate = $\frac{\text{First year CF}}{\text{Equity}}$

5. Valuation After Renovation = $\frac{\text{Stabilised NOI}}{\text{Cap.Rate}}$

6. Valuation After Renovation – PV of loss = Total Value

## Introduction to Commodities and Commodity Derivatives

1. Total Return = Collateral Return (HPY on T-bill) + Price Return $\left(\frac{P_I - P_O}{P_O}\right)$+ Roll Return
2. Price Return = $\frac{\text{Current Price} - \text{Previous Price}}{\text{Previous Price}}$
3. Roll Return = $\frac{\text{Price of Expiring Futures Contract} - \text{Price of New Futures Contract}}{\text{Price of Expiring Futures Contract}}$

# Portfolio Management

## Using Multifactor Models

- **APT** — $E(R_A) = R_f + B_{A,1}\,\lambda_1 + B_{A,2}\,\lambda_2 + \ldots\ldots$
- **Multifactor**
  - Macro-economic: $R_i = E(R_i) + b_1F_1 + b_2F_2 + \ldots. + bnFn + e_t$
  - Fundamental Factor: $R_i = a_i + b_{i,1}F_{P/E} + b_{j,2}\,F_{size} + e_t$
  - Statistical
    - Factor Analysis (co-variance)
    - Principal component (variance)

1. Standardized P/E Sensitivity:
   $b_{i1} = \dfrac{\left(\frac{P}{E}\right)i - \overline{P/E}}{\sigma P/E}$
2. Active Return = $(R_p)$- $(R_B)$
   Active Risk = $\sigma_{(R_P - R_B)}$
3. IR = $\dfrac{\bar{R}_P - \bar{R}_B}{\sigma_{(R_P - R_B)}}$
4. Return Attribution: Active Return = Factor Return + Security Selection Return
5. Factor Return=
   $\sum_{i=1}^{k} (\beta_{Pi} - \beta_{bi}) \times \lambda_i$
6. Active Specific Risk =
   $\sum_{i=1}^{n} (W_{Pi} - W_{bi})^2 \times \sigma_{\varepsilon i^2}$
7. Carhart Model-
   $E(R) = R_F + \beta_1\,RMRF + \beta_2\,SMB + \beta_3\,HML + \beta_4\,WML$

## Measuring and Managing Market Risk

1. $\sigma^2_{Portfolio} = W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2W_AW_BCov_{AB}$
2. E(Ri) = Rf + Betai [E(RMKT) - Rf]
3. Change in Price = - Duration ($\Delta$Y) + $\frac{1}{2}$ Convexity ($\Delta$Y)2
4. Change in Call Price = delta ($\Delta$S) + $\frac{1}{2}$ gamma ($\Delta$S)2 + vega ($\Delta$V)
5. Discount Rate = R + $\pi$ + $\theta$+ $\gamma$ + K + $\emptyset$
6. Inter – temporal Rate of Substitution = $\frac{U_t}{U_O} = \frac{Future}{Current}$
7. PO = E(mt)
8. Real Risk-free rate of Return:
   R = $\frac{1-P_0}{P_0} = \left(\frac{1}{E(m_t)}\right) - 1$
9. $P_o = \frac{E(P_1)}{1+R}$ + cov $(P_1, M_1)$
   $P_o$ is lower; Return ↑ (Since Risk Taken)
   $\frac{E(P_1)}{1+R}$ When no risk = $P_o$
10. Disc. Rate = R+$\pi$ (short term)
    R+$\pi$+ $\theta$ (long- term)
11. Taylor Rule: r = Rn + $\pi$ + 0.5 ($\pi - \pi^*$) + 0.5 (y-y*)
12. BEI = Yield on Non-Inflation Indexed Bond - Yield on Non-Inflation Indexed Bond
13. BEI = $\pi$+ $\theta$

## Analysis of Active Portfolio Management

1. Active Return $E(R_A) = E(R_P) - E(R_B)$
2. For an Active Portfolio of N Securities:
   $E(R_A) = \sum \Delta w_i E(R_i)$
3. Weighted Average of Securities Returns:
   $E(R_P) = \sum w_{P,i} E(R_{P,j})$ and $E(R_B) = \sum w_{B,i} E(R_{B,j})$
4. Ex – ante Active Return:
   $E(R_A) = \sum w_{P,i} E(R_{P,j}) - \sum w_{B,i} E(R_{B,j})$
5. Security Selection Return:
   $E(R_A) = \sum \Delta w_i E(R_{B,j}) + \sum w_{P,i} E(R_{A,j})$
6. Sharpe Ratio = $\frac{R_P - R_F}{\sigma_p}$
7. IR = $\frac{R_P - R_B}{\sigma_{(R_P - R_B)}}$
8. With Optimal Level of Active Risk:
   $SR_P = \sqrt{SR_B^2 + IR^2}$
   Total Risk of The Portfolio: $\sigma_P^2 = \sigma_B^2 + \sigma_A^2$
9. Unconstrained:
   $IR^* = IC \times \sqrt{BR}$
   $E(R_A)^* = IC\sqrt{BR}\sigma_A$
10. Constrained:
    $IR = IC \times \sqrt{BR} \times TC$
    $E(R_A) = IC \times \sqrt{BR} \times TC \times \sigma_A$
    $SR_{pc} = \sqrt{SR_\beta^2 + (IR^2 \times TC)}$
11. $\sigma_{CA} = \frac{T_C \,.\, IR^*}{SR_B} \times \sigma_B$
12. Ex-post Performance Measurement:
    $E(R_A | IC_R) = TC \times IC_R \times \sqrt{BR}\, \sigma_A$
    $R_A = E(R_A | IC_R) + \text{noise}$
13. The Expected Active Return for A Given Target Level of Active Risk: $E(RA) = IR \times \sigma_A$
14. IC = 2(% correct) -1
15. $\sigma_c = [\sigma_x^2 + \sigma_y^2 - 2\, \sigma_x \sigma_y r_{x,y}]$ ½
16. Annualized Active Risk: $\sigma_A = \sigma_c \times \sqrt{BR}$
    Annualized Active Return: $E(R_A) = IC\sqrt{BR} \times \sigma_A$
17. BR = $\frac{N}{1+(N-I)r}$